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HEAT BALANCE-BASED DETERMINATION OF THE CONSUMPTION OF FUEL FOR CONTINUOUS FURNACES

V. B. Kovalevskii, V. N. Papkovich, and S. M. Kozlov

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A solution is considered for the problem of determining fuel consumption in heating a prism of rectangular cross section in a continuous furnace from prescribed temperature conditions.

The amount of fuel needed for maintaining the required temperature conditions in a furnace can be determined on the basis of a heat balance equation [1].

The proposed procedure makes it possible to obtain an equation for calculating instantaneous fuel consumption at the time $0 < t \le t_f$. Unlike the approach described in [1], the present method for solving this problem takes into account not the initial and final temperatures of the metal but rather the instantaneous temperature distribution over the cross section of the ingot, thus increasing substantially the accuracy of the calculations.

Let us consider the given problem using as an example the heating of a prism of rectangular cross section with heat transfer occurring on the surface by means of radiation and convection. According to the assumptions made in [2], the process of heating a metal is described by the system of equations

$$\rho C(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[\lambda(T)\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[\lambda(T)\frac{\partial T}{\partial y}\right], \quad (1)$$

$$0 \le x \le R_1, \quad 0 \le y \le R_2, \quad 0 < t \le t_f,$$

with the initial conditions

$$T(x, y, 0) = T_0$$

and the boundary conditions

$$\lambda(T)\frac{\partial T(R_1, y, t)}{\partial x} = \alpha\left(T_{fur}(t) - T(R_1, y, t)\right) + \sigma\left(T_{fur}^4(t) - T^4(R_1, y, t)\right), \quad (2)$$

$$\frac{\partial T(0, y, t)}{\partial x} = 0,$$

$$\lambda(T)\frac{\partial T(x, R_2, t)}{\partial y} = \alpha\left(T_{fur}(t) - T(x, R_2, t)\right) + \sigma\left(T_{fur}^4(t) - T^4(x, R_2, t)\right), \quad (3)$$

$$\frac{\partial T(x, 0, t)}{\partial y} = 0.$$

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Thus, we assume that the change in the temperature along the Z axis (over the height of the prism) is insignificant and can be neglected. The temperature distribution of the furnace $T_{fur}(t)$ is known for the entire period of heating $0 < t \leq t_f$.

Then the volume-average temperature of the metal can be determined from the formula

$$T_{av}(t) = \frac{1}{\text{mes } V} \iint_V T(x, y, t)\, dxdy . \tag{4}$$

We need to obtain an equation for the fuel consumption at the time $0 < t \leq t_f$.

Let us fix the time t and Δt and consider different expenditures of heat to construct a heat balance equation [1].

1. Heat spent for heating up the metal from $T_{av}(t)$ to $T_{av}(t + \Delta t)$ (useful heat):

$$Q_u(t) = \frac{P}{3600} C_{met} (T_{av}(t + \Delta t) - T_{av}(t)) . \tag{5}$$

2. Heat lost with flue gases leaving the furnace (for fuel-burning furnaces):

$$Q_{out}(t) = B(t) V_{sm} C_{sm} T_{out} \Delta t . \tag{6}$$

3. Heat lost through the lining (laying) of the furnace:

$$Q_{lay}(t) = \frac{T_{fur}(t)}{S_1/\lambda_1 + S_2/\lambda_2 + 1/\alpha_m} F_w \Delta t . \tag{7}$$

4. Heat lost by radiation through open windows and doors of the furnace:

$$Q_h(t) = C_0 (T_{fur}(t)/100)^4 \Phi \psi F \Delta t . \tag{8}$$

5. Heat spent for heating transportation devices (trays, conveyers):

$$Q_{tr}(t) = M_{tr} C_{tr} (T_{tr}(t + \Delta t) - T_{tr}(t)) .$$

We also assume that the change in the heating temperature of the transportation devices corresponds to the change in the temperature of the furnace, i.e.,

$$Q_{tr}(t) = M_{tr} C_{tr} (T_{fur}(t + \Delta t) - T_{fur}(t)) . \tag{9}$$

6. Heat lost through metal rods and inserts in the furnace lining (anchors for fastening a lining made of ceramic fiber, etc.), called thermal lockings:

$$Q_{th.l.}(t) \approx Q_{lay}(t) . \tag{10}$$

7. Heat losses unaccounted for:

$$Q_{un}(t) = 0.1 (Q_{lay}(t) + Q_h(t) + Q_{tr}(t) + Q_{th.l.}(t)) . \tag{11}$$

Thus, the total heat consumption in a continuous fuel-burning furnace at the time t is determined as follows:

$$\Sigma Q_{cons}(t) = Q_u(t) + Q_{out}(t) + Q_{lay}(t) + Q_h(t) + Q_{tr}(t) + Q_{th.l.}(t) + Q_{un}(t) . \tag{12}$$

The heat input is composed of:

1) heat coming from fuel burning (heat of chemical reactions of combustion)

$$Q_{ch.f}(t) = B(t) Q^r_{low} \Delta t , \tag{13}$$

2) heat coming from heating of air supplied to burn the fuel (physical heat of air)

$$Q_{\text{ph.air}}(t) = B(t) V_{\text{air}} C_{\text{air}} T_{\text{air}} \Delta t\,, \tag{14}$$

3) heat coming from fuel heating (physical heat of the fuel)

$$Q_{\text{fuel}}(t) = B(t)\, C_{\text{fuel}} T_{\text{fuel}}\, \Delta t\,. \tag{15}$$

Thus, the total heat input in a continuous fuel-burning furnace at the time t is determined as follows:

$$\Sigma Q_{\text{inp}}(t) = Q_{\text{ch.f}}(t) + Q_{\text{ph.air}}(t) + Q_{\text{ph.f}}(t)\,. \tag{16}$$

According to the energy conservation law, the total consumption of heat should be compensated by its total input into the furnace. Therefore, the heat balance equation has the form

$$\Sigma Q_{\text{cons}}(t) = \Sigma Q_{\text{inp}}(t)\,. \tag{17}$$

Substituting Eqs. (12) and (16) into Eq. (17) and taking into account Eqs. (5)-(11) and (13)-(15), we obtain

$$\begin{aligned} &\frac{P}{3600} C_{\text{met}} \left(T_{\text{av}}(t+\Delta t) - T_{\text{av}}(t)\right) + B(t)\, V_{\text{sm}} C_{\text{sm}} T_{\text{out}} \Delta t + \\ &+ 2.2 \frac{T_{\text{fur}}(t)}{S_1/\lambda_1 + S_2/\lambda_2 + 1/\alpha_{\text{m}}} F_{\text{w}}\, \Delta t + 1.1 C_0 \left(T_{\text{fur}}(t)/100\right)^4 \Phi \psi F \Delta t + \\ &+ 1.1 M_{\text{tr}} C_{\text{tr}} \left(T_{\text{fur}}(t+\Delta t) - T_{\text{fur}}(t)\right) = B(t)\, Q_{\text{low}}^{\text{r}} \Delta t + B(t) V_{\text{air}} C_{\text{air}} T_{\text{air}} \Delta t + B(T)\, C_{\text{fuel}} T_{\text{fuel}}\, \Delta t\,. \end{aligned} \tag{18}$$

From Eq. (4) it follows that

$$\begin{aligned} T_{\text{av}}(t+\Delta t) - T_{\text{av}}(t) &= \frac{4}{\text{mes}\, V} \int_0^{R_1} \int_0^{R_2} \left(T(x, y, t+\Delta t) - \right. \\ &\left. - T(x, y, t)\right) dx dy = \frac{4\Delta t}{\text{mes}\, V} \int_0^{R_1} \int_0^{R_2} \frac{\partial T}{\partial t}(x, y, \theta)\, dx dy\,. \end{aligned} \tag{19}$$

Taking into account Eq. (1) and boundary conditions (2) and (3), we obtain

$$\begin{aligned} &\rho C(T) \int_0^{R_1} \int_0^{R_2} \frac{\partial T}{\partial t}(x, y, \theta)\, dx dy = \\ &= \int_0^{R_1} \int_0^{R_2} \left(\frac{\partial}{\partial x} \left[\lambda(T) \frac{\partial T}{\partial x}(x, y, \theta) \right] + \frac{\partial}{\partial y} \left[\lambda(T) \frac{\partial T}{\partial y}(x, y, \theta) \right] \right) dx dy = \\ &= \int_0^{R_1} \int_0^{R_2} \frac{\partial}{\partial x} \left[\lambda(T) \frac{\partial T}{\partial x}(x, y, \theta) \right] dx dy + \int_0^{R_1} \int_0^{R_2} \frac{\partial}{\partial y} \left[\lambda(T) \frac{\partial T}{\partial y}(x, y, \theta) \right] dx dy = \\ &= \int_0^{R_2} \lambda(T) \frac{\partial T}{\partial x}(x, y, \theta) \Big|_0^{R_1} dy + \int_0^{R_2} \lambda(T) \frac{\partial T}{\partial y}(x, y, \theta) \Big|_0^{R_2} dx = \end{aligned}$$

$$= \int_0^{R_2} [\alpha (T_{fur}(\theta) - T(R_1, y, \theta)) + \sigma (T_{fur}^4(\theta) - T^4(R_1, y, \theta)] \, dy +$$

$$+ \int_0^{R_1} [\alpha (T_{fur}(\theta) - T(x, R_2, \theta)) + \sigma (T_{fur}^4(\theta) - T^4(x, R_2, \theta)] \, dx . \quad (20)$$

Substituting Eq. (20) into Eq. (19), we have

$$T_{av}(t + \Delta t) - T_{av}(t) = \frac{4\Delta t}{\text{mes } V \rho \, C(T)} \times \int_0^{R_2} [\alpha (T_{fur}(\theta) - T(R_1, y, \theta)) +$$

$$+ \sigma (T_{fur}^4(\theta) - T^4(R_1, y, \theta)] \, dy + \int_0^{R_1} [\alpha (T_{fur}(\theta) - T(x, R_2, \theta)) + \sigma (T_{fur}^4(\theta) - T^4(x, R_2, \theta)] \, dx . \quad (21)$$

Substituting Eq. (21) into (18), we combine similar terms, divide them by Δt, and pass to the limit as $\Delta t \to 0$. Taking into account that

$$\lim_{\Delta t \to 0} \frac{T_{fur}(t + \Delta t) - T_{fur}(t)}{\Delta t} = \frac{dT_{fur}(t)}{dt}$$

and

$$\lim_{\Delta t \to 0} T(x, y, \theta) = T(x, y, t) , \quad t \le \theta \le t + \Delta t ,$$

we finally obtain

$$\frac{dT_{fur}(t)}{dt} = A_1 B(t) - A_2 T_{fur}(t) - A_3 T_{fur}^4(t) - A_4 \left(\int_0^{R_2} [\alpha (T_{fur}(t) - T(R_1, y, t) + \right.$$

$$\left. + \sigma (T_{fur}^4(t) - T^4(R_1, y, t)] \, dy + \int_0^{R_1} [\alpha (T_{fur}(t) - T(x, R_2, t)) + \sigma (T_{fur}^4(t) - T^4(x, R_2, t)] \, dx \right) ,$$

where

$$A_1 = \frac{Q_{low}^r + V_{air} C_{air} T_{air} + C_{fuel} T_{fuel} - V_{sm} C_{sm} T_{out}}{1.1 M_{tr} C_{tr}} ;$$

$$A_2 = \frac{2F_w}{1.1 M_{tr} C_{tr} (S_1/\lambda_1 + S_2/\lambda_2 + 1/\alpha_m)} ;$$

$$A_3 = \frac{C_0 \Phi \psi F}{10^8 M_{tr} C_{tr}} ; \quad A_4 = \frac{P C_{met}}{990 \rho C(T) \text{ mes } V M_{tr} C_{tr}} .$$

The proposed procedure for determining the consumption of fuel was tested at the Belarusian Metallurgical Plant in Zhlobin. The experimental results showed satisfactory accuracy of the calculations. The prescribed temperature conditions of operating of a furnace, as well as the fuel consumption calculated from the equation obtained, are given in Fig. 1. In this case the total gas discharge was 500 m^3.

Thus, a procedure for determining fuel consumption was obtained on the basis of a constructed heat balance equation for a continuous furnace. Results of an industrial experiment testify to sufficient efficacy of the proposed

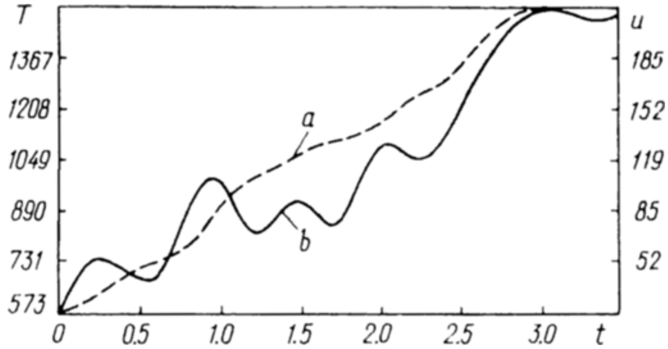

Fig. 1. Diagram of the variation in the temperature of the furnace (a) and in fuel consumption (b). T, K; u, m^3/h.

approach. The sequence of operations proposed in the article can be used to construct equations for calculating fuel consumption for heating problems described by other mathematical models.

NOTATION

t, current time, h; t_f, time of the end of the heating process, h; R_1, R_2, half the length and width of the narrow face of the prism, m; x, y, current coordinates of the narrow face of the prism reckoned from the center, m; $T_{fur}(t)$, temperature of the furnace at the time t, K; σ, coefficient of heat exchange by convection, $W/(m^2 \cdot K)$; δ, coefficient of heat exchange by radiation, $W/(m^2 \cdot K^4)$; $\lambda(T)$, thermal conductivity, $J/(m \cdot h \cdot K)$; $C(T)$, heat capacity, $J/(kg \cdot K)$; ρ, density of the material, kg/m^3; T_0, initial, uniform temperature distribution in the prism, K; $T(x, y, t)$, temperature at the point (x, y) at the time t, K; mes V, cross-sectional area of the prism, m^2; P, furnace efficiency, kg/h; C_m, mean specific heat of the metal, $J/(kg \cdot K)$; $T_{av}(t)$ and $T_{av}(t + \Delta t)$, volume-average temperatures at the times t and Δt, K; $B(t)$, fuel consumption, kg/sec, m^3/sec; V_{sm}, volume of the combustion products formed in burning 1 kg or 1 m^3 of fuel, m^3/kg, m^3/m^3; C_{sm}, specific heat of the combustion products, $J/(m^3 \cdot K)$; T_{out}, temperature of outgoing flue gases (adopted in accordance with the temperature conditions of the furnace); T_{amb}, temperature of the ambient air, K; S_1/λ_1 and S_2/λ_2, thermal resistances (ratios of the thickness of the lining layers to their thermal conductivity coefficients) for the first and second layers, $m^2 \cdot K/W$; α_m, coefficient of heat transfer from the external surface of the furnace walls to the ambient, $W/(m^2 \cdot K)$; F_w, area of the external surface of the furnace lining, m^2; $C_0 = 5.7\ W/(cm^2 \cdot K^4)$, radiation factor of a blackbody; Φ, diaphragming coefficient (determined from the diagram given in [1] on page 49); ψ, relative time of opening of a window or small door (if the window is open 30 min in 1 h, then $\psi = 0.5$); F, area of the window or small door, m^2; M_{tr} and C_{tr}, mass (kg) and mean specific heat ($J/(kg \cdot K)$) of the transportation devices located in the furnace in the time Δt, respectively; $T_{tr}(t)$ and $T_{tr}(t + \Delta t)$, temperature of the transportation devices at the times t and $t + \Delta t$, respectively, K; Q^r_{low}, lowest heat of fuel combustion, J/kg, J/m^3; V_{air}, volume of the air needed for burning 1 kg or 1 m^3 of fuel (with account for the required excess air), m^3/kg, m^3/m^3; C_{air}, mean specific heat of the air, $J/(cm^3 \cdot K)$; T_{air}, temperature of air heating, K; C_{fuel}, mean specific heat of the fuel, $J/(kg \cdot K)$, $J/(m^3 \cdot K)$; T_{fuel}, temperature of fuel heating, K.

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